Name _____ Date _____ Class _____

LESSON 12-5

Practice A

Mathematical Induction and Infinite Geometric Series

Determine whether the geometric series converges or diverges.

1. $25 + 20 + 16 + 12.8 + \ldots$

   a. Find and simplify the common ratio. _____

   b. Use the ratio to determine whether the series converges or diverges. _____

2. $\frac{1}{16} + \frac{1}{8} + \frac{1}{4} + \frac{1}{2} + \cdots$ _____

3. $35.2 + 8.8 + 2.2 + 0.55 + \ldots$ _____

Find the sum of each infinite geometric series, if it exists.

4. $864 + 576 + 384 + 256 + \ldots$

   a. Test the series for convergence. _____

   b. Use the formula $S = \frac{a_1}{1 - r}$ to find the sum. _____

5. $200 + 150 + 112.5 + 84.375 + \ldots$ _____

6. $\frac{2}{5} - \frac{4}{25} + \frac{8}{125} - \frac{16}{625} + \cdots$ _____

7. $\frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \frac{16}{625} + \cdots$ _____

8. $\frac{16}{625} - \frac{8}{125} + \frac{4}{25} - \frac{2}{5} + \cdots$ _____

Write each repeating decimal as a fraction in simplest form.

9. $0.\overline{57}$

   a. Write the decimal as an infinite series.

   _____

   b. Find the common ratio. _____

   c. Use $S = \frac{a_1}{1 - r}$ to write the fraction. _____

10. $0.\overline{7}$ _____

11. $0.8\overline{3}$ _____

12. $0.\overline{23}$ _____

3. 2, 4, 6, 9 or $2, \frac{1}{4}, -\frac{3}{2}, 9$

4. When $d = 0$, $a_2 = 2$.; When $d = 1$, $a_2 = 3$.

5. $x = \frac{ac - b^2}{2b - c - a}$.

6. Possible answer: Arithmetic mean is $\frac{a^2 + b^2}{2}$. Geometric mean is ab or $-ab$.

   Show: $\frac{a^2 + b^2}{2} \geq ab$ and $\frac{a^2 + b^2}{2} \geq -ab$

   Since all squares are nonnegative, $(a - b)^2 \geq 0$; so $a^2 - 2ab + b^2 \geq 0$ and $\frac{a^2 + b^2}{2} \geq ab$. Since all squares are nonnegative, $(a + b)^2 \geq 0$; so $a^2 + 2ab + b^2 \geq 0$ and $\frac{a^2 + b^2}{2} \geq -ab$.

Problem Solving

1. a. $a_n = a_1 (r^{n-1})$

   b. $a_1 = 100$; because all the sunlight reaches layer 1

   c. $r = 0.9$; r represents the fraction of sunlight that passes through a layer.

   d. $a_7 = 100(0.9)^{(7-1)}$; 53%

   e. 15 layers

2. a. 18%

   b. 7 layers

3. D

4. H

Reading Strategy

1. Possible answer: Find the ratio of each term to the previous term.

2. a. $\frac{1}{3}$

   b. 5

   c. $\frac{1}{10}$

   d. $\frac{1}{4}$

3. Possible answer: The terms of geometric sequences have common ratios; the terms of arithmetic sequences have common differences.

4. a. Exponential.

   b. Linear.

5. a. $\frac{1}{3}$

   b. $a_1 = 18$

   c. $\frac{2}{81}$

LESSON 12-5

Practice A

1. a. $\frac{4}{5}$

   b. Converges

2. Diverges

3. Converges

4. a. Converges

   b. 2592

5. 800

6. $\frac{2}{7}$

7. $\frac{2}{3}$

8. Does not exist

9. a. $0.57 + 0.0057 + 0.000057 + \ldots$

   b. 0.01

   c. $\frac{19}{33}$

10. $\frac{7}{9}$

11. $\frac{5}{6}$

12. $\frac{23}{99}$

Practice B

1. Diverges

2. Converges

3. $\frac{28}{3}$

4. 312.5

5. Does not exist

6. ≈ -30.46

7. $\frac{16}{99}$

8. $\frac{16}{999}$

9. $\frac{16}{990}$

10. $\frac{1}{22}$

11. $\frac{1}{9}$

12. $\frac{41}{333}$

13. Possible answer: $n = -5$