Name ________________________ Date ______________ Class______________

LESSON 6-5

Practice B

Finding Real Roots of Polynomial Equations

Solve each polynomial equation by factoring.

1. $9x^3 - 3x^2 - 3x + 1 = 0$

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2. $x^5 - 2x^4 - 24x^3 = 0$

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3. $3x^5 + 18x^4 - 21x^3 = 0$

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4. $-x^4 + 2x^3 + 8x^2 = 0$

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Identify the roots of each equation. State the multiplicity of each root.

5. $x^3 + 3x^2 + 3x + 1 = 0$

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6. $x^3 + 5x^2 - 8x - 48 = 0$

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Identify all the real roots of each equation.

7. $x^3 + 10x^2 + 17x = 28$

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8. $3x^3 + 10x^2 - 27x = 10$

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Solve.

9. An engineer is designing a storage compartment in a spacecraft. The compartment must be 2 meters longer than it is wide and its depth must be 1 meter less than its width. The volume of the compartment must be 8 cubic meters.

   a. Write an equation to model the volume of the compartment.

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   b. List all possible rational roots. ________________________

   c. Use synthetic division to find the roots of the polynomial equation. Are the roots all rational numbers?

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   d. What are the dimensions of the storage compartment? ______________

8. $x^3 - 1^3$
   $(x - 1)(x^2 + x + 1)$

Challenge

1. $\{[(6)x + 8]x - 5\}x + 1$
2. $(\{[(-2)x + 5]x - 1\}x + 3)x - 4$
3. $N(3) = 128$
4. $N(3) = 128$
5. $N(x) = 5x^3 - 3x^2 + 7x - 1$
6. $N(3) = 5 \cdot 3^3 - 3 \cdot 3^2 + 7 \cdot 3 - 1 = 128$
7. $P(x) = \{[(2)x - 6]x - 2\}x - 30$, $P(4) = \{[(2)4 - 6]4 - 2\}4 - 30 = [(2)4 - 2]4 - 30 = (6)4 - 30 = -6$; The numbers in the innermost nests are the coefficients of the quotient, and the last number is the remainder. So the quotient is $2x^2 + 2x + 6$ and the remainder is –6.
8. The quotient is $2x^2 + 2x + 6$ and the remainder is –6.

Problem Solving

1. a. –2, 4, 6
   b. $(x + 2)(x - 4)(x - 6)$
2. a. –4, –1, 0
   b. $(2x + 2)(x + 4)(x)$
3. a. 5, 8, 9
   b. $(x - 5)(x - 8)(x - 9)$
4.

Basket	Dimensions (in terms of *x*)	Actual Dimensions	Volume
A	$(x - 5)$, $(x - 8)$, $(x - 9)$	7 by 4 by 3	84 cubic units
B	$(x + 2)$, $(x - 4)$, $(x - 6)$	14 by 8 by 6	672 cubic units
C	$(2x + 2)$, $(x + 4)$, (x)	26 by 16 by 12	4992 cubic units

5. No; the dimensions of each basket are doubled from one size to the next except for 14 to 26.
6. No; $\dfrac{84}{672} \neq \dfrac{672}{4992}$

Reading Strategies

1. Multiply $(x + 3)$ and $(x^2 + 2)$.
2. No; there are no two factors that have $x^2 + 2$ as their product.
3. 5
4. a. $(x^3 - 8x^2) + (-x + 8)$
   b. x^2
   c. –1
   d. $x^2(x - 8) - 1(x - 8) = (x - 8)(x^2 - 1)$
   e. $x^2 - 1$; $(x + 1)(x - 1)$
   f. $(x - 8)(x + 1)(x - 1)$

LESSON 6-5

Practice A

1. $x^2 + 2x + 1$; –1, 0
2. $2x^2 - 18$; –3, –2, 3
3. –5, 0
4. –1, 0, 7
5. $x = -3$ with multiplicity 1; $x = 0$ with multiplicity 1; $x = 1$ with multiplicity 1
6. $x = 0$ with multiplicity 1; $x = 1$ with multiplicity 2
7. $x = 1$ with multiplicity 2; $x = 5$ with multiplicity 1
8. $x = -6$ with multiplicity 2; $x = 0$ with multiplicity 1
9. $\pm\dfrac{1}{3}, \pm 1$
10. ±1, ±2, ±3, ±4, ±6, ±12
11. $\pm\dfrac{1}{2}, \pm 1, \pm 2, \pm\dfrac{5}{2}, \pm 5, \pm 10$
12. ±1, ±3, ±9
13. Stefan is correct. The roots of the expression are 2 and –2, both of which have multiplicity 1.

Practice B

1. $\dfrac{1}{3}, \dfrac{\sqrt{3}}{3}, -\dfrac{\sqrt{3}}{3}$
2. –4, 0, 6
3. –7, 0, 1
4. –2, 0, 4
5. $x = -1$ with multiplicity 3
6. $x = 3$ with multiplicity 1; $x = -4$ with multiplicity 2
7. –4, 1, –7
8. $-5, -\dfrac{1}{3}, 2$
9. a. $x^3 + x^2 - 2x - 8 = 0$

b. $\pm 1, \pm 2, \pm 4, \pm 8$

c. 2, $\frac{-3 \pm i\sqrt{7}}{2}$; no, 2 of the roots are irrational numbers.

d. 2 m wide, 4 m long, and 1 m deep

Practice C

1. $-5, 0, 7$
2. $0, 3, 4$
3. $x = 2$ with multiplicity 3
4. $x = -4$ with multiplicity 2; $x = -2$ with multiplicity 1
5. $-8, 0, 6$
6. $3, 6, 2 \pm \sqrt{3}$
7. $-3, 0, 1$
8. $-3, 1, -3 \pm \sqrt{11}$
9. a. $2x^3 - 4x^2 - 64 = 0$

   b. $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32, \pm 64$

   c. $4, -1 \pm i\sqrt{7}$; no, 2 of the roots are irrational numbers.

   d. 4 in. wide, 8 in. long, and 2 in. deep

Reteach

1. $3x^4(x - 5)(x + 2)$; $-2, 0, 5$
2. $x^2(x^2 - 5x + 6)$; $x^2(x - 2)(x - 3)$; $0, 2, 3$
3. $2x(x^2 - 3x - 18)$; $2x(x - 6)(x + 3)$; $-3, 0, 6$
4. $2x^4(x^2 - 16)$; $2x^4(x + 4)(x - 4)$; $-4, 0, 4$
5. a. $\pm 1, \pm 3, \pm 5, \pm 15$

   b. 3 or 5

$\frac{p}{q}$	**Coefficients of the Equation**			
	1	–7	7	15
1	1	6	13	28
3	1	–4	–5	0
5	1	–2	–3	0

   c. $(x - 3)(x^2 - 4x - 5) = 0$; $(x - 3)(x - 5)(x + 1) = 0$

   d. $x = 3$ or $x = 5$ or $x = -1$

Challenge

1. $y = (x + 3)(x)(x - 4)$
2. $y = -(x + 1)^2(x - 1)(x - 3)$
3. $y = (x+5)(x+2)^2\left(x - \frac{1}{2}\right)(x-3)$
4. $y = (x + 6)^2(x)(x - 1)(x - 3)^2(x - 4)^2$
5. $y = (x+3i)(x-3i)(x-1)^2\left(x - \left(1+\sqrt{3}\right)\right)\left(x + \left(1+\sqrt{3}\right)\right)$

Problem Solving

1. $V = w(w + 10)(w - 14)$
2. $w^3 - 4w^2 - 140w - 76{,}725 = 0$
3. No; yes; no

   The constant term is 76,725, which is not a multiple or 4 or 10, but is a multiple of 5.
4. Students should test possible roots that are multiples of 5 but not multiples of 10, such as 35, 45, and 55.
5. C
6. A

Reading Strategies

1. Substitute the value of the root in the function and see if it equals 0.
2. $(x - 3)$ and $(x + 2)$
3. a. $(x + 4)$

   b. 3 times
4. $4x$, $(x - 3)$, $(x + 3)$; $-3, 0, 3$
5. $-x$, $(x - 5)$, $(x - 1)$; $0, 1, 5$
6. $(x + 2)$, $(x + 2)$, $(x - 2)$; $-2, 2$

LESSON 6-6

Practice A

1. 3
2. 5
3. 4
4. a. $P(x) = x(x + 1)(x - 2)$

   b. $P(x) = (x^2 + x)(x - 2)$

   c. $P(x) = x^3 - 2x^2 + x^2 - 2x$

   d. $P(x) = x^3 - x^2 - 2x$
5. $P(x) = x^3 - 3x^2 - 13x + 15$
6. $P(x) = x^3 + 4x^2 - x - 4$
7. a. 2

   b. $2i, -2i$