I. Background Information and Problem

Junior Achievement (JA) is an educational program available worldwide. JA uses hands-on experiences to help young people understand the economics of life. In partnership with businesses and educators, JA brings the real world to students. The JA Company Program provides basic economic education for high school students. The Program uses support and guidance of volunteer consultants from the local business community. By organizing and operating an actual business, students learn how businesses function. They also learn about the structure of the free enterprise system and the benefits it provides.

Gates Williams is the production manager for Computer Flips, a Junior Achievement company. Computer Flips purchases a basic computer at wholesale prices and then adds a display, extra memory cards, extra USB ports, or a CD-ROM or DVD-ROM drive. The company also purchases these extra components at wholesale prices. The computers, with the added features, are then resold at retail prices.

Computer Flips produces two models: Simplex and Omniplex. The profit on each Simplex is $200, and on each Omniplex, the profit is $300. The Simplex model has fewer add-ons, so it requires only 60 minutes of installation time. The Omniplex has more add-ons and requires 120 minutes of installation time. Five JA students do all of the installation work. Each of them works 8 hours per week. Mr. Williams must decide the rate of production per week of each computer model in order to maximize the company's weekly profit.

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II. Exploring the problem

To make decisions such as the one Gates Williams faces, operations researchers use a technique known as ***linear programming***. Answering the following questions will help you understand this technique.

One way to approach the problem is to make some guesses and test the profit generated by each guess. Suppose Mr. Williams decides the company should make 20 of each model in one week.

1. How much profit would be generated?

2. How much time is required to install add-ons for 20 Simplex and 20 Omniplex computers?

3. How much total time is available to install add-ons?

4. a) Is there enough installation time to produce 20 Simplex and 20 Omniplex computers?
   b) Why or why not?

1. ____________________

2. ____________________

3. ____________________

4a. Yes No

4b. ____________________

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II. (continued) Exploring the problem

5. If Gates Williams decides to make the following number of computers, answer Question (1) and Question (4) based upon the new information.

    a) 10 Simplex and 26 Omniplex computers

    b) 26 Simplex and 10 Omniplex computers

6. a) Is it possible to find a production mix for which there is enough installation time?
    b) If so, give an example.
    c) If you answered part (b), how much profit do the production rates you found generate for the company?

5a. Profit: ____________

Yes No

____

5b. Profit: ____________

Yes No

____

6a. Yes No

6b. ____________

6c. Profit: ____________

III. Generalizing the problem

Sometimes, it is helpful to visualize things. The numbers, variables, and their relationships in a problem can be represented by a graph. Before graphing the Computer Flips problem, you must translate the information in the problem into mathematical statements—equations or inequalities.

Let x_1 = the weekly production rate of Simplex computers (i.e. the number of Simplex computers made in a week), and x_2 = the weekly production rate of Omniplex computers (i.e. the number of Omniplex computers made in a week).

The variables x_1 and x_2 are called ***decision variables***. Gates Williams will use them to help make his decision. Mr. Williams goal is to make as much money as possible. How does he do this? By selling as many computers as he is able. Therefore, Mr. Williams can calculate his weekly profit, p, as a function of x_1 and x_2. Because the objective is to maximize profit, the profit function is called the ***objective function***.

7. Write an equation for the profit, P, the company would earn in a week. [Hint: Look back at Question (1) and see how you calculated profit for 20 Simplex and 20 Omniplex computers.]

8. a) Can Computer Flips produce a negative number of either computer model?
   b) Write two mathematical statements that describe your answer to part (a).

9. Write a mathematical statement that describes the relationship between the installation times required to produce x_1 Simplex and x_2 Omniplex computers each week and the amount of available installation time each week. [Hint: Look back at Question (2) and Question (3) and see if you had enough installation time for 20 Simplex and 20 Omniplex computers.]

7. ______________________________

8a. __________ Yes __________ No __________

8b. ______________________________

9. ______________________________

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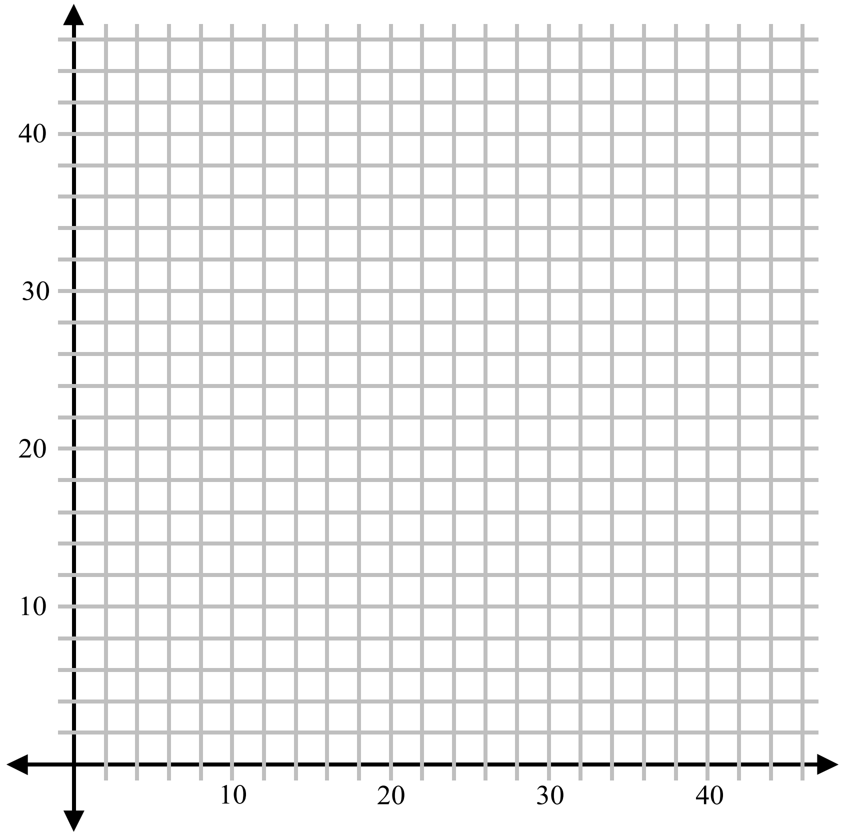
IV. A visual approach

10. On the same coordinate axes, graph the three *inequalities* you have written for Question (8b) and Question (9). For uniformity, place x_1 on the horizontal axis and x_2 on the vertical axis. Make sure to identify clearly the common region.



11. Give one point that satisfies all three inequalities.

12. What is the connection between the shaded region on the graph and the context of the problem?

13. a) Plot the points (20, 20), (10, 26), and (26, 10) on the coordinate plane in Question (10).

b) Which of these points satisfy all three inequalities?

c) These points correspond to the computer quantities in Question (1) and Question (5). How does their location on the graph relate to your answers in Question (4) and Question (5)?

10. See Work

11. ________________

12. See Work

13a. See Work

13b. ________________

13c. See Work

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V. The linear programming process

Based on your answers in section IV, it should be clear that Gates Williams cannot make whatever number of computers he chooses because there is limited amount of installation time available each week. More formally, we say that the available installation time ***constrains*** the number of Simplex and Omniplex computers that can be made each week. Therefore, we call the inequality that captures this relationship a ***constraint***. Since the other two inequalities express the fact that the *decision variables* (x_1 and x_2) in this problem cannot be negative, they are called the ***non-negativity constraints***.

All the points that satisfy the constraint inequalities represents a mix of Simplex and Omniplex computers that could be produced each week. We call this region of the coordinate system the ***feasible region*** because those points represent feasible (possible) production mixes.

14. Choose any point in the feasible region and compute the weekly profit that would be generated by producing that mix of Simplex and Omniplex computers. Recall, you determined the profit function in Question (7).

15. Choose a second point in the feasible region that generates the *same* weekly profit as the first point. Draw a line through the two points on the graph in Question (10).

    Every point on the line you have drawn generates the same weekly profit. For this reason, such a line is called a ***line of constant profit***.

16. Repeat the process in Questions (14) and (15) with two new points. Choose two random points in the feasible region and compute the weekly profit for each. Now find a companion point for each that generates the same weekly profit, and draw the two new lines of constant profit.

17. What do you notice about the three lines you have drawn?

18. Which of the points generates the largest weekly profit?

19. Where does the line that generates the largest possible weekly profit intersect the feasible region? What is the profit at that point?

14. Point: ____________

    Profit: ____________

15. See Work ____________

16. Point: ____________

    Profit: ____________

    Point: ____________

    Profit: ____________

17. See Work ____________

18. Point: ____________

    Profit: ____________

19. Point: ____________

    Profit: ____________

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VI. Solving the problem.

Hopefully, section V (page 6) has suggested to you that the point or points representing the largest possible weekly profit are close to the boundary of the feasible region.

20. Choose a point on the boundary of the feasible region, but not at a corner, and evaluate the profit there.

21. Continue to choose points on the boundary, but try to increase the amount of profit each time.

Point	Profit

22. Finally, evaluate the profit at each corner point of the feasible region.

Point	Profit

23. What is the relationship between the corner points and the set of constraints?

24. a) At which of the points Questions (21) and (22) is the profit the greatest?
 b) How would you describe this point? Note: The point with the maximum profit is called the ***optimal solution***.

25. If a linear programming problem has a unique optimal solution, where in the feasible region do you think it occurs?

20. Point: ____________

Profit: ____________

21. See Work ____________

22. See Work ____________

23. See Work ____________

24a. ____________

24b. See Work ____________

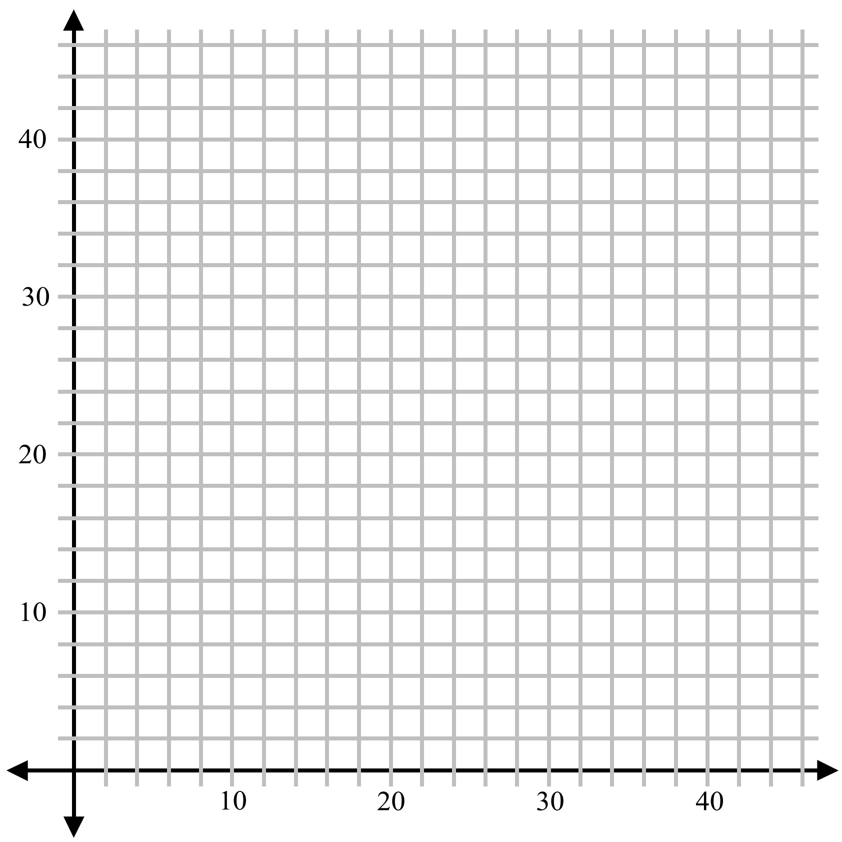
25. See Work ____________

VII. Complicating the problem

After several weeks of operation, one of the students in the sales department of Computer Flips does some market research. Based on this research, she decides that the company cannot sell more than 20 Simplex computers or more than 16 Omniplex computers in a week.

26. Write two mathematical statements that express these market constraints.

27. Graph the new system of constraint inequalities. Look back at Question (8) and Question (9) for the other constraint inequalities. For uniformity, place x_1 on the horizontal axis and x_2 on the vertical axis. Make sure to identify clearly the feasible region.



28. What do you notice about the optimal solution you found in Question (24)?

29. What is the optimal solution after adding the market constraint?

26. ______________________

27. See Work

28. See Work

29. Point: ______________________

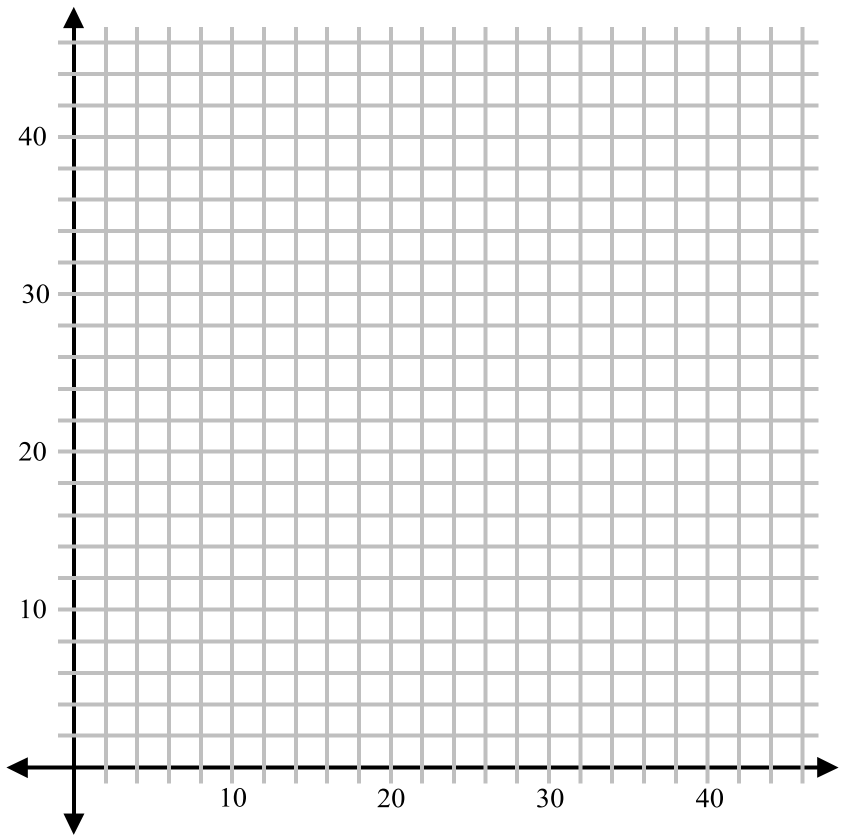
Profit: ______________________

VIII. Complicating the problem even more

The students at Computer Flips notice that they are getting many returns. Every computer that was returned had a problem with one of the add-ons. They realize that they need to test their finished products before shipping them. The students decide to assign the task of testing the computers to only one of the student installers. To accommodate this change, the other four student installers agree to work 10 hours per week, so that the total available installation time remains 40 hours per week, or 2,400 minutes. The student who does the testing also works 10 hours per week. It takes her 20 minutes to test a Simplex computer and 24 minutes to test an Omniplex computer.

30. Write a mathematical statement that expresses this testing constraint.

31. Graph the new system of constraint inequalities. Look back at Question (8), Question (9), and Question (26) for the other constraint inequalities.



32. How does the new constraint change the feasible region?

33. Is your solution in Question (29) still optimal? If not, what is the optimal solution after adding the testing constraint?

30. __________

31. See Work

32. See Work

33. Point:

Profit:

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IX. Even more complications? Adding more variables

Computer Flips has some initial success, so the students are considering producing two additional models: Multiplex and Megaplex. Multiplex will have more add-ons than Simplex, but not as many as Omniplex. Each Multiplex will generate $250 profit. Megaplex, as the name implies, will have more add-ons than any of the other models. Each Megaplex will generate $400 profit.

34. What are the decision variables in the new problem? [Hint: Look back to section III.]

35. Write an equation for the profit, *p*, the company would earn in a week. [Hint: Look back at Question (7) to see how you calculated profit for just Simplex and Omniplex computers.]

The installation and testing times for each Multiplex and Megaplex computer appear in Table 1. In addition, market research indicates that the *combined* sales of Simplex and Multiplex cannot exceed 20 computers per week, and the *combined* sales of Omniplex and Megaplex cannot exceed 16 computers per week.

	Simplex	Omniplex	Multiplex	Megaplex
Installation Time	60 min.	120 min.	90 min.	150 min.
Testing Time	20 min.	24 min.	24 min.	30 min.

Table 1. Installation and Testing times for all four computer models.

36. Using the information in Table 1, formulate the constraints after the Multiplex and Megaplex models have been added to the product mix. [Hint: Look back at Question (8b), Question (9), Question (26), and Question (30) to see how you represented the constraints for just Simplex and Omniplex computers.]

37. Is it possible to solve this problem by graphing? Why or why not?

34. See Work ______________________

35. ______________________

36. See Work ______________________

37. See Work ______________________